Training session: dominating sets, clique-width

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1. Prove or disprove: if G is a graph with $n \geq 2$ vertices, then $\gamma(G) < n$.
2. Prove that if G is a graph without isolated vertices, then $\gamma(G) \leq |V(G)|/2$.
3. A dominating set in a graph G is **minimal** if it does not contain any other dominating set.

   Prove or disprove: any two minimal dominating sets of the n-vertex cycle, C_n, have the same cardinality.
4. Prove that the clique-width of an n-vertex path, P_n, is at most 3.
5. Prove that the clique-width of any tree T is at most 3.
6. Let G be a split graph with its vertex set partitioned into a clique C and independent set I. Show that if x and y are two distinct vertices in C such that $N[x] \subseteq N[y]$, then $\gamma(G) = \gamma(G - x)$.
7. Let G be a connected graph with $n \geq 3$ vertices. Show that G has a connected dominating set with at most k vertices if and only if G has a spanning tree with at least $n - k$ leaves.
8. Show that any two efficient dominating sets in a graph G have the same cardinality.
9. For a connected graph G, let $\gamma_c(G)$ denote the **connected domination number** of G, that is, the smallest cardinality of a dominating set that induces a connected graph. Clearly, $\gamma(G) \leq \gamma_c(G)$. Is there a function f such that every connected graph G satisfies $\gamma_c(G) \leq f(\gamma(G))$?